

Solar system constraints on $f(T)$ gravity

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ABSTRACT

We use recent observations from solar system orbital motions in order to constrain $f(T)$ gravity. In particular, imposing a quadratic $f(T)$ correction to the linear-in- T form, which is a good approximation for every realistic case, we extract the spherical solutions of the theory. Using these spherical solutions to describe the Sun's gravitational field, we use recently determined supplementary advances of planetary perihelia, to infer upper bounds on the allowed $f(T)$ corrections. We find that the maximal allowed divergence of the gravitational potential in $f(T)$ gravity from that in the teleparallel equivalent of General Relativity is of the order of 6.2×10^{-10} , in the applicability region of our analysis. This is much smaller than the corresponding (significantly small too) divergence that is predicted from cosmological observations, as expected. Such a tiny allowed divergence from the linear form should be taken into account in $f(T)$ model building.

Key words: gravitation–relativistic processes–celestial mechanics–ephemerides

1 INTRODUCTION

According to accumulating observations of different kinds, the universe is now known to be accelerating (Riess et al.(1998), Perlmutter et al. (1999)). Such a feature led physicists to follow two directions in order to explain it. The first one is to introduce the concept of dark energy (see Copeland et al. (2006) and Bamba et al. (2012a) and references therein) in the right-hand-side of the field equations of General Relativity, which could either be the simple cosmological constant or various new exotic ingredients (Ratra & Peebles (1988), Wetterich (1988), Caldwell (2002), Nojiri & Odintsov (2003), Feng et al.(2005), Guo (2005), Cai et al. (2010)). The second direction is to modify the left-hand-side of the general relativistic field equations, that is to modify the gravitational theory itself, with the basic extended gravitational theories known as $f(R)$ -gravity (see (De Felice (2010)) and references therein). Such extended theories can present very interesting behaviors; the corresponding cosmologies have been investigated in detail (Nojiri & Odintsov (2007)).

A new class of modified-gravity theories, which has recently received attention by the scientific community, is the $f(T)$ gravity (Ferraro & Fiorini (2007), Ferraro & Fiorini (2008), Bengochea & Ferraro (2009), Linder (2010)). It is an extension of the old idea of the “teleparallel” equivalent of General Relativity (TEGR), in which one uses the curvature-less Weitzenböck connection instead of the torsion-less Levi-Civita one, and where the dynamical objects are the four linearly independent vierbeins (Einstein (1928), Unzicker & Case (2005), Hayashi & Shirafuji (1979). In TEGR,

the torsion tensor is formed solely from products of first derivatives of the tetrad. Then, the Lagrangian density T can be constructed from such a torsion tensor under the assumptions of invariance under general coordinate transformations, global Lorentz transformations, and the parity operation, along with requiring the Lagrangian density to be second order in the torsion tensor (Hayashi & Shirafuji (1979)). Thus, in $f(T)$ gravity one generalizes the aboveTEGR formalism, making the Lagrangian density a function of T , similar to the $f(R)$ extension of Einstein-Hilbert action.

The above $f(T)$ -gravitational framework proves to lead to interesting cosmological behavior and it has gained much attention in the literature (Myrzakulov (2011), Yerzhanov et al (2010), Wu & Yu (2010a), Chen et al. (2011), Wu & Yu (2011a), Bamba et al. (2010), Dent et al. (2011), Zheng & Huang (2011), Bamba et al. (2011), Yang (2011), Zhang et al. (2011), Cai et al. (2011), Chattopadhyay & Debnath (2011), Sharif & Rani (2011), Wei et al. (2011a), Ferraro & Fiorini (2011a), Wei et al. (2011b), Capozziello et al. (2011), Wu & Yu (2011b), Bamba & Geng (2011), Geng et al. (2011), Wei (2012), Geng et al (2012), Wu & Geng (2011), Bohmer et al. (2012), Karami & Abdolmaleki (2011), Atazadeh & Darabi (2012), Farajollahi et al. (2012), Yang et al. (2012), Karami & Abdolmaleki (2012a), Karami & Abdolmaleki (2012b), Xu et al (2012), Bamba et al. (2012b), Setare & Houndjo (2012), Liu et al. (2012), Wu & Yu (2010b), Bengochea (2011), Gonzalez et al. (2012), Daouda et al. (2011), Daouda et al. (2012a), Boehmer et al (2011), Daouda et al. (2012b), Ferraro & Fiorini (2011c), Wang (2011), Miao et al. (2011), Wei et al (2012), Ferraro & Fiorini (2011b)). One question that straightforwardly arises is what are the qualitative and quantitative $f(T)$ -modifications that are allowed. Although theoretically one has an enhanced freedom, in practice, in order to confront observations one expects a small divergence from

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TEGR, that is from the linear-in- T case which coincides with General Relativity. Indeed, in Wu & Yu (2010b) and Bengochea (2011) the authors used data from cosmological observations in order to constrain the model parameters of two well-studied $f(T)$ -ansatzes, namely the power-law and the exponential one, and they found less than 1% divergence fromTEGR.

In the present work we use latest observations of solar system orbital motions in order to constrain the parameters of $f(T)$ gravity. In particular, after extracting the spherical solutions of the theory we use them to describe the Sun's gravitational field and then we can use data from planetary motion in order to infer an upper bound on the allowed divergence from theTEGR. The solar system analysis imposes significantly tighter bounds than the cosmological ones (Wu & Yu (2010b), Bengochea (2011)) as expected, and in this completely different observational region it verifies too that the divergences fromTEGR (and thus from General Relativity) are very small.

The plan of the work is as follows: In section 2 we briefly review $f(T)$ gravity and in section 3 we extract the spherical solutions assuming small corrections to the linear-in- T scenario. In section 4 we use data from solar system orbital motions and we impose constraints on the model parameters. Finally, in section 5 we summarize the obtained results.

2 $F(T)$ GRAVITY

In this section we briefly review $f(T)$ gravity. Our notation is as follows: Greek indices μ, ν, \dots and capital Latin indices A, B, \dots run over all coordinate and tangent space-time 0, 1, 2, 3, while lower case Latin indices (from the middle of the alphabet) i, j, \dots and lower case Latin indices (from the beginning of the alphabet) a, b, \dots run over spatial and tangent space coordinates 1, 2, 3, respectively.

As stated in the Introduction, the dynamical variable of “teleparallel” gravity, as well as of its $f(T)$ extension, is the vierbein field $e_A(x^\mu)$. This forms an orthonormal basis for the tangent space at each point x^μ of the manifold, that is $e_A \cdot e_B = \eta_{AB}$, where $\eta_{AB} = \text{diag}(1, -1, -1, -1)$. Furthermore, the vector e_A can be analyzed with the use of its components e^μ_A in a coordinate basis, that is $e_A = e^\mu_A \partial_\mu$.

In such a construction, the metric tensor is obtained from the dual vierbein as

$$g_{\mu\nu}(x) = \eta_{AB} e^\mu_A(x) e^\nu_B(x). \quad (1)$$

Contrary to General Relativity, which uses the torsion-less Levi-Civita connection, in the present formalism one uses the curvature-less Weitzenböck connection $\Gamma^{\lambda(w)}_{\mu\nu} \equiv e^\lambda_A \partial_\mu e^\nu_A$ (Weitzenböck (1923)), whose torsion tensor reads

$$T^\lambda_{\mu\nu} = \Gamma^{\lambda(w)}_{\nu\mu} - \Gamma^{\lambda(w)}_{\mu\nu} = e^\lambda_A (\partial_\mu e^\nu_A - \partial_\nu e^\mu_A). \quad (2)$$

Moreover, the contorsion tensor, which equals to the difference between Weitzenböck and Levi-Civita connections, is defined as $K^{\mu\nu}_\rho \equiv -\frac{1}{2} \left(T^{\mu\nu}_\rho - T^{\nu\mu}_\rho - T_\rho^{\mu\nu} \right)$ and we also define $S_\rho^{\mu\nu} \equiv \frac{1}{2} \left(K^{\mu\nu}_\rho + \delta^\mu_\rho T^{\alpha\nu}_\alpha - \delta^\nu_\rho T^{\alpha\mu}_\alpha \right)$.

Using these quantities one can define the teleparallel Lagrangian, which is the torsion scalar, as (Hayashi & Shirafuji (1979), Maluf (1994), Arcos & Pereira (2004))

$$T \equiv S_\rho^{\mu\nu} T^\rho_{\mu\nu}. \quad (3)$$

In summary, in the present formalism all the information concerning the gravitational field is included in the torsion tensor $T^\lambda_{\mu\nu}$, and

the torsion scalar T arises from it in a similar way as the curvature scalar arises from the curvature (Riemann) tensor.

While in the teleparallel equivalent of General Relativity (TEGR) the action is just T , the idea of $f(T)$ gravity is to generalize T to a function $T + f(T)$, which is similar in spirit to the generalization of the Ricci scalar R in the Einstein-Hilbert action to a function $f(R)$. In particular, the action in a universe governed by $f(T)$ gravity reads:

$$I = \frac{1}{16\pi G} \int d^4x e [T + f(T) - 2\Lambda + L_m], \quad (4)$$

where $e = \det(e^\mu_A) = \sqrt{-g}$, G is the Newton's constant (we also set for convenience the light speed to one), L_m stands for the matter Lagrangian, and we have added for completeness a cosmological constant Λ (which could alternatively be absorbed into $f(T)$). We mention here that since the Ricci scalar R and the torsion scalar T differ only by a total derivative (Weinberg (2008)), in the case where $f(T)$ is zero the action (4) is equivalent to General Relativity with a cosmological constant.

Variation of the action (4) with respect to the vierbein gives the equations of motion

$$e^{-1} \partial_\mu (e e^\rho_A S^{\mu\nu}_\rho) [1 + f_{,T}] - e^\lambda_A T^\rho_{\mu\lambda} S^{\mu\nu}_\rho + e^\rho_A S^{\mu\nu}_\rho \partial_\mu (T) f_{,TT} - \frac{1}{4} e^\lambda_A [T + f(T) - 2\Lambda] = 4\pi G e^\rho_A T^{\nu(em)}_\rho, \quad (5)$$

where $f_{,T}$ and $f_{,TT}$ denote respectively the first and second derivatives of the function $f(T)$ with respect to T . Note that the tensor $T^{\nu(em)}_\rho$ on the right-hand side is the usual energy-momentum tensor.

3 SPHERICALLY SYMMETRIC SOLUTIONS

We are interested in extracting spherically symmetric solutions of the four-dimensional $f(T)$ gravity presented in the previous section, and in particular to extract the correction to the Schwarzschild solution of General Relativity. Up to now in the literature it has been done in three dimensions (Gonzalez et al. (2012)) and partially in four dimensions for specific $f(T)$ ansatzes (Daouda et al. (2011), Daouda et al. (2012a), Boehmer et al. (2011), Daouda et al. (2012b)), while there is also a different approach, namely to find the general (non-diagonal) vierbein choice that corresponds exactly to the Schwarzschild solution (Ferraro & Fiorini (2011b), Ferraro & Fiorini (2011c), Wang (2011), Maluf et al. (2012)).

Let us consider as usual the general metric ansatz of the form

$$ds^2 = N(r)^2 dt^2 - K(r)^{-2} dr^2 - R(r)^2 d\Omega^2, \quad (6)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ and where $N(r)$, $K(r)$ and $R(r)$ are three unknown functions. Note that we do not restrict in the case $N(r) = K(r)$ as in (Daouda et al. (2012b)), since we desire to be as general as possible. As a next step we need to extract the corresponding vierbeins that give rise to the above metric, through relation (1). The simplest choice is to use a diagonal vierbein of the form

$$e^\mu_A = \text{diag} (N(r), K(r)^{-1}, R(r), R(r) \sin \theta). \quad (7)$$

However, we mention that although in the case of linear-in- T gravity, such a simple relation between the metric and the vierbeins is always allowed, in the general $f(T)$ gravity this is not the case anymore, and in general one has a more complicated relation connecting the vierbein tetrad with the metric, with the former being non-diagonal even for a diagonal metric (Sotiriou et al. (2011)), as

it arises from Lorentz transformations of (7). Nevertheless, in the 4D cosmological investigations of $f(T)$ gravity (Ferraro & Fiorini (2007), Ferraro & Fiorini (2008), Bengochea & Ferraro (2009), Linder (2010), Myrzakulov (2011), Yerzhanov et al (2010), Wu & Yu (2010a), Chen et al. (2011), Wu & Yu (2011a), Bamba et al. (2010), Dent et al. (2011), Zheng & Huang (2011), Bamba et al. (2011), Yang (2011), Zhang et al. (2011), Cai et al. (2011), Chatopadhyay & Debnath (2011), Sharif & Rani (2011), Wei et al. (2011a), Ferraro & Fiorini (2011a), Wei et al. (2011b), Capozziello et al. (2011), Wu & Yu (2011b), Bamba & Geng (2011), Geng et al. (2011), Wei (2012), Geng et al (2012), Wu & Geng (2011), Bohmer et al. (2012), Karami & Abdolmaleki (2011), Atazadeh & Darabi (2012), Farajollahi et al. (2012), Yang et al. (2012), Karami & Abdolmaleki (2012a), Karami & Abdolmaleki (2012b), Xu et al (2012), Bamba et al. (2012b), Setare & Houndjo (2012), Liu et al. (2012), Wu & Yu (2010b), Bengochea (2011), Ferraro & Fiorini (2011c), Daouda et al. (2011), Daouda et al. (2012a)), as well as in its black hole solutions (Wang (2011), Miao et al. (2011), Wei et al (2012), Ferraro & Fiorini (2011b)), the authors still use the simple relation between the vierbeins and the metric, as a first approach to reveal the structure and the features of the theory. Therefore, in the present work, we also assume the simple relation between the vierbeins and the metric as a first approach on this novel theory, capable of revealing the main features of the solutions. Clearly a detailed investigation of the general vierbein choice in $f(T)$ gravity, and its relation to extra degrees of freedom, is a necessary step for the understanding of this new theory (Cai et al. (2012)).

Inserting the vierbein choice (7) into the torsion scalar definition (3) we obtain

$$T(r) = 2K^2 \frac{R'}{R} \left(\frac{2N'}{N} + \frac{R'}{R} \right), \quad (8)$$

where primes denote derivatives with respect to r . Thus, substitution in the field equations (5), neglecting the matter energy-momentum tensor, provides the following separate equations of motion:

$$0 = K^2 \frac{R'}{R} T' f_{TT} - \left(\frac{T + f - 2\Lambda}{4} \right) + \left\{ T - \frac{1}{R^2} - K^2 \left[\left(\frac{2N'}{N} - \frac{2K'}{K} \right) \frac{R'}{R} - 2 \frac{R''}{R} \right] \right\} \left(\frac{1 + f_T}{2} \right), \quad (9)$$

$$0 = \left(\frac{1}{R^2} - T \right) \left(\frac{1 + f_T}{2} \right) + \left(\frac{T + f - 2\Lambda}{4} \right), \quad (10)$$

$$0 = -K^2 \left(\frac{N'}{N} + \frac{R'}{R} \right) T' f_{TT} + \left(\frac{T + f - 2\Lambda}{4} \right) - \left\{ \frac{T}{2} + K^2 \left[\frac{R''}{R} + \frac{N''}{N} - \frac{N'^2}{N^2} + \left(\frac{N'}{N} + \frac{R'}{R} \right) \left(\frac{N'}{N} + \frac{K'}{K} \right) \right] \right\} \left(\frac{1 + f_T}{2} \right). \quad (11)$$

Finally, we mention here that while in teleparallel gravity, as well as in General Relativity, the off-diagonal components of the field equations vanish, in $f(T)$ gravity even the diagonal vierbein field gives rise to an extra equation (Boehmer et al. (2011)). In particular, the r - θ equation reads:

$$\frac{K \cot \theta}{2R^2} T' f_{TT} = 0. \quad (12)$$

The above equations (8)-(12) cannot obtain analytical solutions in general. Thus, we proceed making two assumptions. The first is the usual one in order to reduce the unknown functions, namely without loss of generality we set $R(r) = r$ (going to more

general $R(r)$ will only lead to more complicated expressions but qualitatively the results will be similar). The second assumption is related to the $f(T)$ ansatz. In particular, since the usual General Relativity is obtained by setting $f(T)$ to zero, it is expected that a realistic $f(T)$ must be small comparing to T , and as we mentioned this has been verified using cosmological observations (Wu & Yu (2010b), Bengochea (2011)). Therefore, by expanding every $f(T)$ ansatz in T -powers, we deduce that as a first approximation we can consider a form of

$$f(T) = \alpha T^2 + O(T^3), \quad (13)$$

where we do not write the constant and the linear terms in the above expansion since they can be respectively absorbed in Λ and T terms of the action (4). In the above expression α is a pure $f(T)$ -parameter that determines at first approximation the divergence from teleparallel gravity, that is from General Relativity. We mention here that in Friedmann-Robertson-Walker metric $T = -6H^2$ (Linder (2010)), while in the Schwarzschild metric $T \propto \Lambda$ for large r (Aldrovandi & Pereira (2010), Ferraro & Fiorini (2011b)), that is in both cases $T \ll 1$ in S.I units, which is an additional indication that the above expansion is justified (moreover that is why we do not consider ansatzes of negative power laws of T). Finally, the above ansatz has been used as a first non-linear correction of $f(T)$ gravity in (Ferraro & Fiorini (2011b), Gonzalez et al. (2012), Daouda et al. (2012b)). In summary, going beyond the assumption (13), although it would bring significant mathematical complications, it would not qualitatively change our results.

Thus, under the ansatz (13), equations (9)-(12) up to $O(\alpha/r^2)^2$ lead to the solution:

$$N(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 + \alpha \left[-6\Lambda - \frac{6}{r^2} - \frac{4GM\Lambda}{c^2 r} \right] \quad (14)$$

$$K(r)^2 = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda}{3} r^2 + \alpha \left[\frac{8\Lambda}{3} - \frac{14}{r^2} - 2\Lambda^2 r^2 - \frac{2GM}{c^2 r} \left(8\Lambda - \frac{8}{r^2} \right) \right], \quad (15)$$

where we have restored the light speed, and where we have introduced the integration constant GM , with M the mass of the spherical object. We mention here that in the above solutions we have kept only up to linear terms in α/r^2 , since higher-order terms are expected to be extremely small for r in the region of the solar system that we have orbital data. In any case, this assumption will be checked a posteriori, after we extract the bounds on α . Finally, we straightforwardly observe that in the limit $\alpha \rightarrow 0$, the above solution coincides with the Schwarzschild-de Sitter one (Rindler (2006)) as expected, while for $\alpha, \Lambda \rightarrow 0$ it becomes the usual Schwarzschild one.

4 SOLAR SYSTEM CONSTRAINTS

In the previous section we extracted the spherically symmetric solutions of $f(T)$ gravity, in the case where there is a small but general divergence from the T -gravity, that is from the teleparallel equivalent of General Relativity (TEGR). Therefore, in the present section we apply these solutions to the gravitational field of the Sun, and we use the latest solar system data from planetary orbital motions (Fienga (2011)) to perform a sensitivity analysis on the two model parameters, namely the cosmological constant Λ and the parameter α that determines the divergence from General Relativity. Finally,

as usual we neglect the effects of spatial curvature, which is a robust approximation for solar system scales.

As usual, for a gravitational field determined by a spherically symmetric metric $g_{\mu\nu}$, the Newtonian potential and its corrections are extracted from the 00 component, namely if $g_{00} = 1 + h_{00}$, where h_{00} is dimensionless, then the potential is $U = h_{00}c^2/2$. Therefore, from the g_{00} form of $f(T)$ gravity spherical solutions, that is from $N(r)^2$ of (14), we observe that there are two corrections to the Newtonian potential U_N affecting a test particle's orbit, namely

$$U_\Lambda = -\frac{\Lambda c^2 r^2}{6} \quad (16)$$

$$U_\alpha = -\frac{3\alpha c^2}{r^2}. \quad (17)$$

We mention here that the term $-6\alpha\Lambda$ is uniform, that is it does not depend on r , and therefore it does not affect orbital motions, while the term $\propto \alpha\Lambda GM/r$ does not yield secular orbital precessions, since it just corresponds to a Newtonian term with GM rescaled. Thus, in the following it is adequate to consider only the terms (16),(17). Lastly, note that the dimensions of the two parameters are

$$[\Lambda] = \text{L}^{-2}, \quad (18)$$

$$[\alpha] = \text{L}^2. \quad (19)$$

The orbital effects of extra-potentials having the functional forms of (16),(17) have been incorporated analytically several times with a variety of different approaches in the framework of solar system investigations (Islam (1983), Cardona & Tejeiro (1998), Kerr et al. (2003), Kraniotis & Whitehouse (2003), Jetzer & Sereno (2006), Kagramanova et al. (2006), Sereno & Jetzer (2006), Adkins et al. (2007b), Adkins et al. (2007a), Sereno & Jetzer (2007), Iorio (2006), Iorio (2008), Iorio (2012)), and they can be straightforwardly handled using, for instance, the standard Lagrange perturbative scheme (Bertotti et al. (2003)). From the equations for the variations of the osculating Keplerian orbital elements (Bertotti et al. (2003)) we immediately deduce that only the longitude of the pericenter $\varpi \doteq \Omega + \omega$ (with Ω the longitude of the ascending node and ω the argument of pericenter) and the mean anomaly \mathcal{M} undergo secular precessions, due to the spherical symmetry of (16),(17) and their time-independence. On the other hand, the semi-major axis a , the eccentricity e , the inclination I of the orbital plane to the reference $\{x, y\}$ plane chosen, and the longitude of the ascending node Ω , remain unaffected¹. However, from the point of view of a comparison with solar system observations, we are interested only in the secular precessions of the perihelia, since the rates of the mean anomaly are dominated by the further, additive mis-modeling in the Keplerian mean motions $n_b \doteq \sqrt{GM}a^{-3/2}$ due to the uncertainty $\sigma_{GM} = 10 \text{ km}^3 \text{ s}^{-2}$ in the solar gravitational parameter GM (Konopliv et al. (2011)).

The Lagrange equation for the secular precession of the longitude of pericenter is (Bertotti et al. (2003)):

$$\left\langle \frac{d\varpi}{dt} \right\rangle = -\frac{1}{n_b a^2} \left\{ \left[\frac{(1-e^2)^{1/2}}{e} \right] \frac{\partial \langle \mathcal{R} \rangle}{\partial e} + \frac{\tan(I/2)}{(1-e^2)^{1/2}} \frac{\partial \langle \mathcal{R} \rangle}{\partial I} \right\}, \quad (20)$$

¹ Indeed, the Lagrange rate equation for a (Bertotti et al. (2003)) contains the partial derivative of the averaged perturbing potential $\langle U_{\text{pert}} \rangle$ with respect to \mathcal{M} , which is proportional to t . The Lagrange equations for e, I, Ω (Bertotti et al. (2003)) are formed with the partial derivatives of $\langle U_{\text{pert}} \rangle$ with respect to Ω, ω, I (and \mathcal{M} as well in the Lagrange equation for e), which are absent in spherically symmetric perturbing potentials.

Table 1. Supplementary rates of change $\Delta\dot{\Omega}$ and $\Delta\dot{\varpi}$, of nodes Ω and perihelia ϖ respectively, of some planets of the solar system, in milliarcseconds per century (mas cty^{-1}), estimated by Fienga et al. (2011) with the INPOP10a ephemerides. Data from Messenger and Cassini were included for Mercury and Saturn. The reference $\{x, y\}$ plane is the mean Earth's equator at J2000.0.

	$\Delta\dot{\Omega} \left(\frac{\text{mas}}{\text{cty}} \right)$	$\Delta\dot{\varpi} \left(\frac{\text{mas}}{\text{cty}} \right)$
Mercury	1.4 ± 1.8	0.4 ± 0.6
Venus	0.2 ± 1.5	0.2 ± 1.5
Earth	0.0 ± 0.9	-0.2 ± 0.9
Mars	-0.05 ± 0.13	-0.04 ± 0.15
Jupiter	-40 ± 42	-41 ± 42
Saturn	-0.1 ± 0.4	0.15 ± 0.65

where

$$\mathcal{R} = \langle U_{\text{pert}} \rangle \quad (21)$$

is the average of the perturbing potential over one full orbital revolution of the test particle. Inserting (16),(17) into (20) we obtain

$$\dot{\varpi}_\Lambda = \frac{1}{2} \left(\frac{\Lambda c^2}{n_b} \right) \sqrt{1-e^2}, \quad (22)$$

$$\dot{\varpi}_\alpha = \frac{3\alpha c^2}{a^4 n_b (1-e^2)}, \quad (23)$$

where from now on for conciseness we neglect the brackets $\langle \dots \rangle$ denoting the average over one orbital period, while the overdots indicate the time derivative.

In order to preliminarily² infer upper bounds on both Λ and α , we compare (22),(23) for some planets to the supplementary rates of change $\Delta\dot{\varpi}$ of their perihelia ϖ , listed in Table 1, determined from observations by Fienga et al. (2011) with the INPOP10a ephemerides.

However, one must be careful in straightforwardly comparing (22),(23) to the extra-rates of Table 1, since $\Delta\dot{\varpi}$, by construction, account for all those competing forces which were not modeled³ in the software of the INPOP10a ephemerides. Moreover, $\Delta\dot{\varpi}$ are also affected by the unavoidable mismodeling in the effects which were actually included in the models fitted to the observations in INPOP10a. Thus, one should correct $\Delta\dot{\varpi}$ for those unmodeled/mismodeled dynamical features of motion, whose magnitude is at least comparable to the uncertainties in determining $\Delta\dot{\varpi}$ themselves. In this respect, the actions of the Sun's quadrupole mass moment J_2 (Rozelot & Damiani (2011)), currently known with a 10% uncertainty, and the general relativistic Lense-Thirring effect (Lense & Thirring (1918)), caused by the Sun's angular momentum S and not modeled at all in INPOP10a, are relevant since they induce competing secular perihelion precessions whose size is large

² An alternative approach would consist of explicitly modeling $f(T)$ gravity in the planetary data processing softwares of the ephemerides, and fitting such ad-hoc modified dynamical models to the same observations. Thus, α and Λ would be estimated as solve-for parameters.

³ We only deal with well known and established forces, both Newtonian and Einsteinian, since it would be pointless to invoke the potentially disturbing action of any sort of putative, exotic effect, whose alleged biasing action may, after all, be present even if a full covariance analysis is performed, as outlined in footnote 2.

enough, as far as the inner planets, whose extra-rates are more accurately determined, are concerned. The explicit analytical expressions, valid for a generic spatial orientation of the Sun's rotational axis (actually it does not point exactly to the North Celestial Pole), of the perihelion precessions due to J_2 and the Lense-Thirring effect, can be found in Iorio (2011).

Having at our disposal the extra-precessions of the perihelia of more than one planet, it is possible to set up linear combinations of the form

$$\dot{\omega}_A = \Lambda k_A^{(\Lambda)} + \alpha k_A^{(\alpha)} + J_2 k_A^{(J_2)} + S k_A^{(LT)}, \quad (24)$$

A = Mercury, Venus, ...,

where the coefficients $k_A^{(\dots)}$ come from the analytical expressions of the precessions due to the various effects considered similar to (22),(23). In summary, it is possible to construct a non-homogeneous linear system of four equations for the four unknowns Λ, α, J_2, S , which is solvable. The resulting values of Λ and α are, by construction, independent of both J_2 and S , whatever their values may be. By propagating the uncertainties in $\Delta\dot{\omega}_A$ entering the expressions of Λ and α , it is possible to preliminarily constrain them. By using the first four inner planets, it turns out that

$$|\Lambda| \leq 6.1 \times 10^{-42} \text{ m}^{-2}, \quad (25)$$

$$|\alpha| \leq 1.8 \times 10^4 \text{ m}^2. \quad (26)$$

It may be noticed that the bound on Λ of (25) is tighter than the one in Iorio (2006) by about one order of magnitude.

Since we have now extracted the upper bound on the $f(T)$ -correction allowed from solar system orbital motions, we can examine the accuracy of the approximations made in our analysis. Firstly, taking r to be the Mercury mean heliocentric distance $\bar{r}_{\text{Merc}} = a(1 + \frac{e^2}{2}) = 5.9 \times 10^{10} \text{ m}$ (a is its semimajor axis and e its eccentricity), which is the smallest distance where we have observational data, we obtain $\alpha/\bar{r}_{\text{Merc}}^2 \approx 5 \times 10^{-18}$, and this justifies our approximation to neglect orders of $\mathcal{O}(\alpha/r^2)$ in (14),(15). Additionally, given the small value of the cosmological constant, we indeed see that the two cross-terms of (14), which are proportional to $\alpha\Lambda$, that we did not take into account in the analysis, are many orders of magnitude smaller than the terms (16),(17) that we did use (whether we use Mercury's data, that is enhancing the α -term, or the Pluto's data, that is enhancing the Λ -term), and thus our approximation is well-justified. Moreover, calculating T from (8) we find that $T \approx -2\Lambda(1 + 6\alpha\Lambda)$ for $r \gg 1$ (in S.I units), a result that verifies our indication that $T \ll 1$ (in S.I units) in (13). Finally, we can also verify that $\alpha T^2 \ll T$, that is the correction to the linear term is very small as expected.

Relation (26) is the main result of the present work, that is it is the upper bound of the $f(T)$ -correction to teleparallel gravity (that is to General Relativity), allowed from solar system orbital motions. In order to provide this allowed correction in a more transparent way, we denote by $\Delta U_{f(T)}$ the divergence of the gravitational potential in $f(T)$ gravity from that in the teleparallel equivalent of General Relativity, defined as the ratio of the correction term $6|\alpha|r^{-2}$ in (14) to the standard term $2GMc^{-2}r^{-1}$. The maximal allowed value of $\Delta U_{f(T)}$ will be acquired at the Mercury's orbit $r = \bar{r}_{\text{Merc}}$, in which the correction term is the largest one, since \bar{r}_{Merc} is the smallest distance in the region where data exist. Thus, we find

$$\Delta U_{f(T)} \Big|_{r=\bar{r}_{\text{Merc}}} \approx \frac{6|\alpha|\bar{r}_{\text{Merc}}^{-2}}{2GMc^{-2}\bar{r}_{\text{Merc}}^{-1}} \lesssim 6.2 \times 10^{-10}, \quad (27)$$

as expected the allowed $f(T)$ -correction affecting solar system orbital motions is very small.

5 CONCLUSIONS

In this work we used latest data from solar system orbital motions to constrain $f(T)$ gravity. In particular, considering the basic and usual ansatz $f(T) = \alpha T^2$, which is a good approximation in all realistic cases, and including also a cosmological constant Λ for completeness, we extracted the spherically symmetric solutions of the theory, which coincide with the Schwarzschild-de Sitter one in the limit $\alpha \rightarrow 0$. Thus, by describing the Sun's exterior gravitational field by these solutions, we were able to use data from planetary motions in order to constrain α and Λ .

Concerning the cosmological constant Λ , we obtained the usual tiny bounds. Interestingly enough, our current $f(T)$ -analysis leads to one order of magnitude tighter constraints than General Relativity (2006), that is without considering the α -term in the metric. Concerning the pure $f(T)$ -parameter α , the obtained bound (26) leads to a maximal allowed divergence of the gravitational potential in $f(T)$ gravity from that in the teleparallel equivalent of General Relativity of the order of $\lesssim 6.2 \times 10^{-10}$ for the smallest distance \bar{r}_{Merc} (Mercury mean heliocentric distance) that we have data, and where the correction term is the largest one.

The above obtained small divergence from General Relativity is much smaller than the corresponding (significantly small too) divergence that is predicted from cosmological constraints (Wu & Yu (2010b), Bengochea (2011)), where one also finds that $f(T)$ must be close to the linear-in- T form. This is standard in observational constraints and justified, since the solar system observations are always more accurate than the cosmological ones. In summary, in the present analysis we did verify, in a different context, the expected result that the allowed divergences of $f(T)$ gravity from the linear (teleparallel) form are significantly small, and this should be taken into account in $f(T)$ model building.

In the above analysis we remained in the diagonal vierbein ansatz, since in this case one can safely elaborate the spherical solutions. However, observing the features of some specific spherical solutions under non-diagonal ansatzes (Daouda et al. (2012b)), in which one obtains similar terms with the present expressions, we deduce that the above analysis would lead to qualitatively similar results even in those cases. However, since for the moment it is not clear how to elaborate the non-diagonal cases and in particular how to handle the extra degrees of freedom at the background and especially at the perturbation level, we preferred to remain in the diagonal vierbein scenario.

We close this work by mentioning that since in the solutions of the aforementioned analysis there appear terms of the form α/r^2 (which are small for planetary motions as discussed above and even a full Parametrized-Post-Newtonian (PPN) analysis (Will (2005)) will not change the obtained results), they could be significant at much smaller distances, outside the applicability region of the present analysis. Thus, an interesting and necessary investigation would be to constrain the allowed correction of $f(T)$ -gravity using different scenarios like, for example, fast extrasolar planets orbiting their parent stars at distances smaller than Mercury. Furthermore, Earth-based laboratory experiments (in which the simple spherical geometry will not be the only case), where the allowed corrections are also expected to be small according to the recent tests on General Relativity (Dimopoulos et al. (2007), Turyshv (2008)), would be worthwhile. However, what could happen in even smaller dis-

tances is unknown, where the possibility of large divergences from General Relativity could remain open.

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